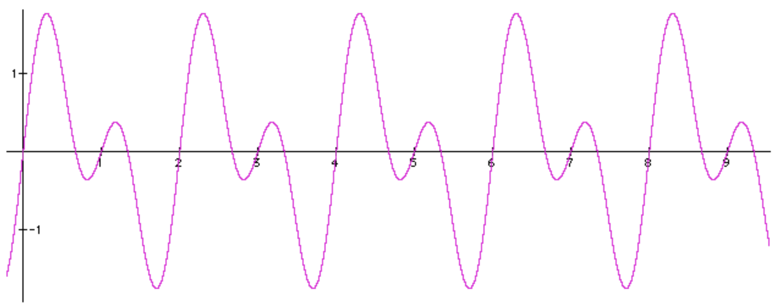
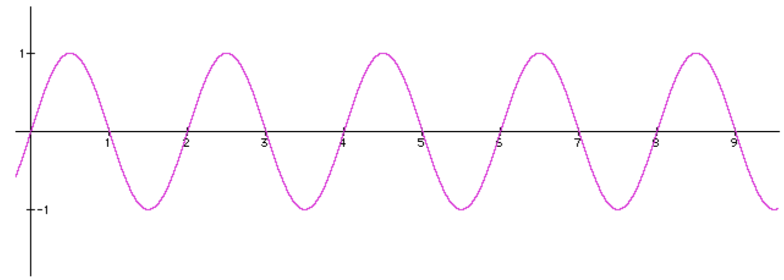
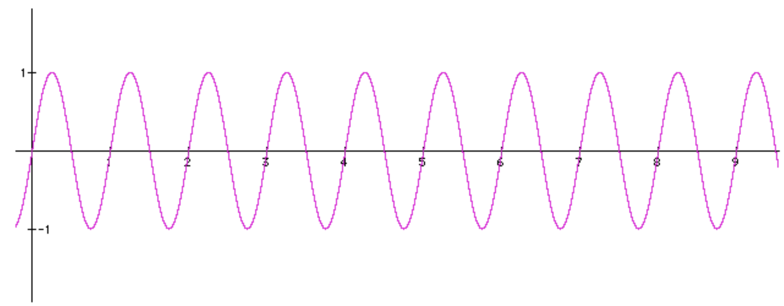
Introduction to Fourier Analysis

- Definition of Fourier analysis
- Analysis of periodic waves
- Analysis of aperiodic waves
- Time-frequency uncertainty
- Digitization and spectrograms
- Reading focus:
  - pp. 41-68
  - boxes 3.2 and 3.3 but not 3.1
  - Raven manual, appendices A and B

The Fourier series



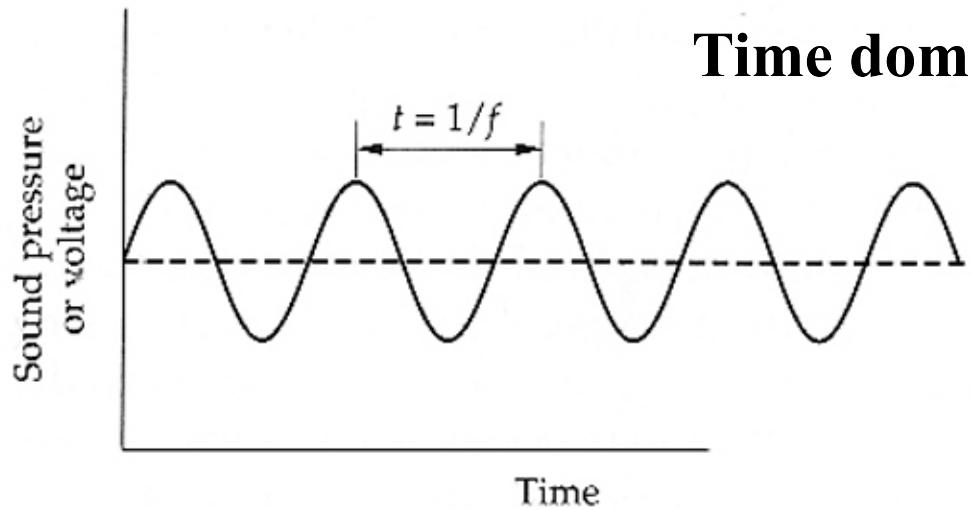




- Any continuous waveform can be partitioned into a sum of sinusoidal waves (Fourier is a type of infinite series)
- $P(t) = P_o + \sum P_n \, \mathit{sin}\,(2\pi f_n t + \Phi_n)$
- P_o is the ambient pressure
- P_n is the pressure of the nth sine wave
- f_n is the frequency of the nth sine wave
- Φ_n is the phase of the nth sine wave

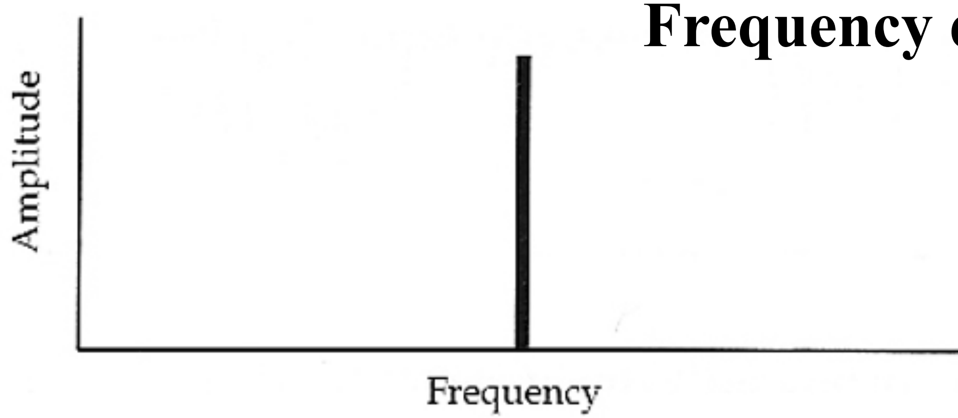
Sound spectrum

Time domain



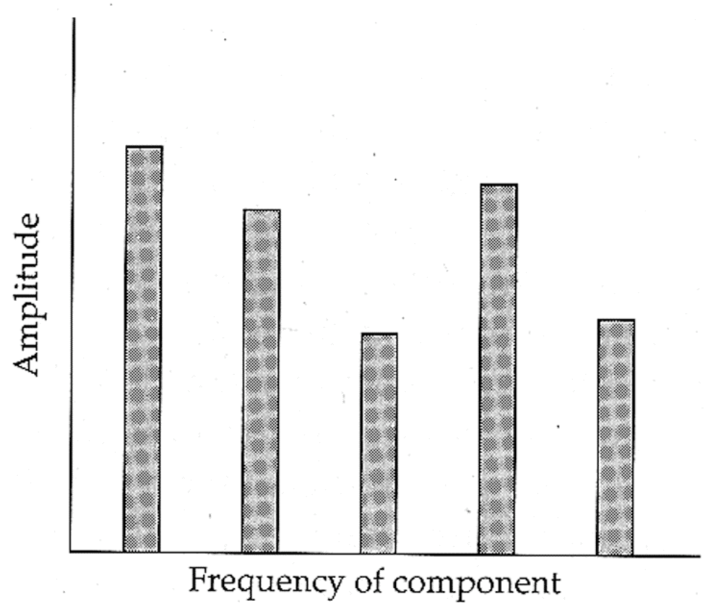
Frequency domain

Frequency spectrum

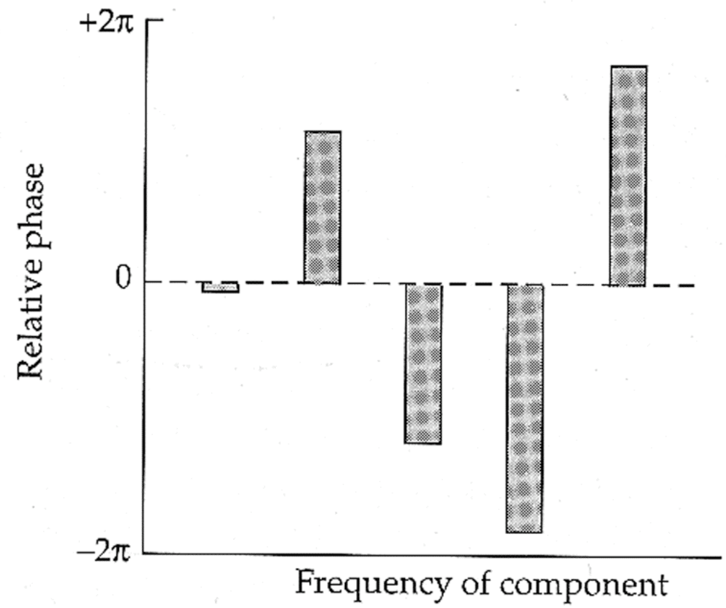


Frequency domain of a complex wave

Frequency spectrum



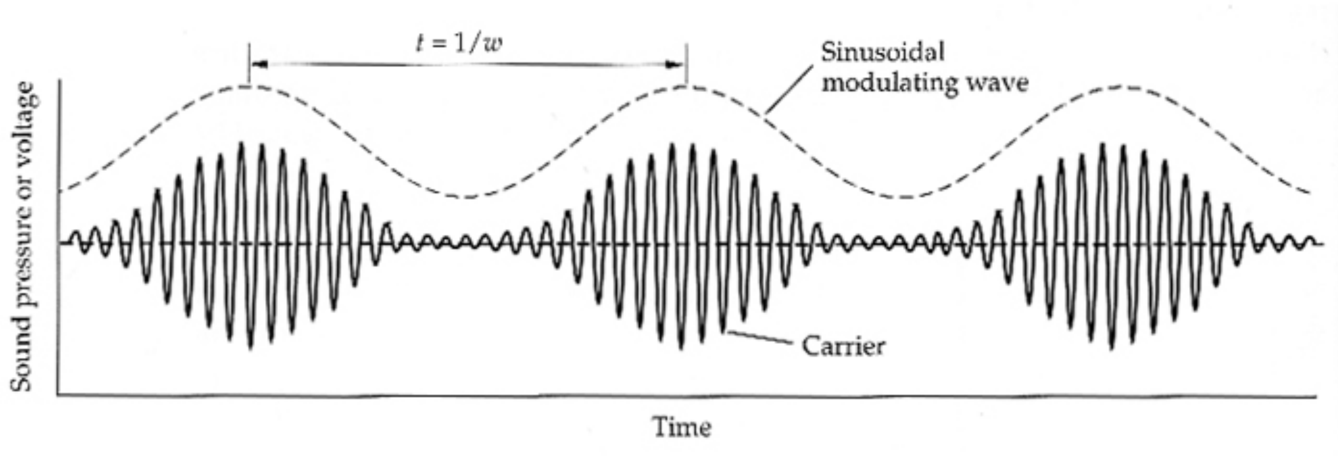
Phase spectrum



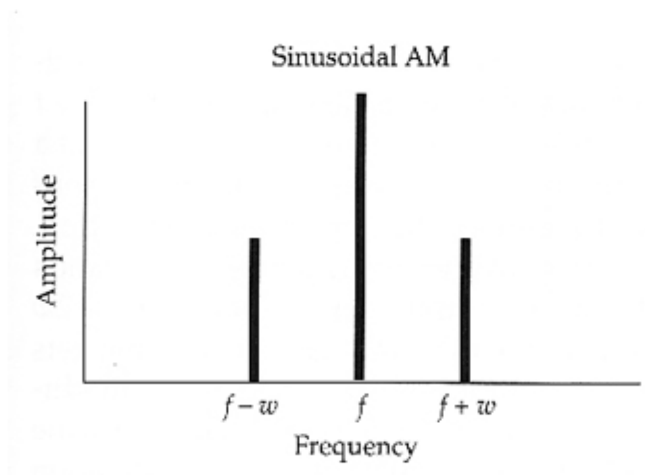
Types of periodic waveforms

- Amplitude varies in a repeating manner - amplitude modulation
- Frequency varies in a repeating manner - frequency modulation
- The repeating shape of the waveform is not a sine wave - nonsinusoidal periodic wave

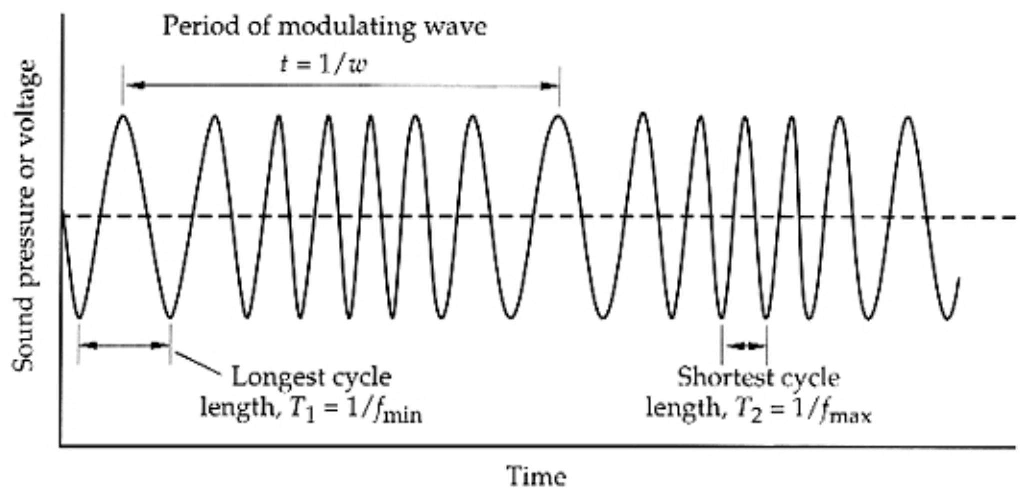
Amplitude modulation (AM)



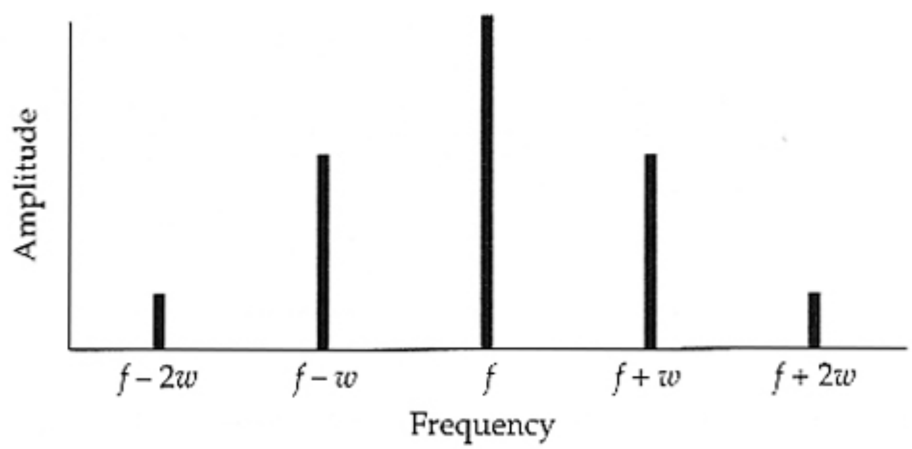
Carrier frequency plus side bands



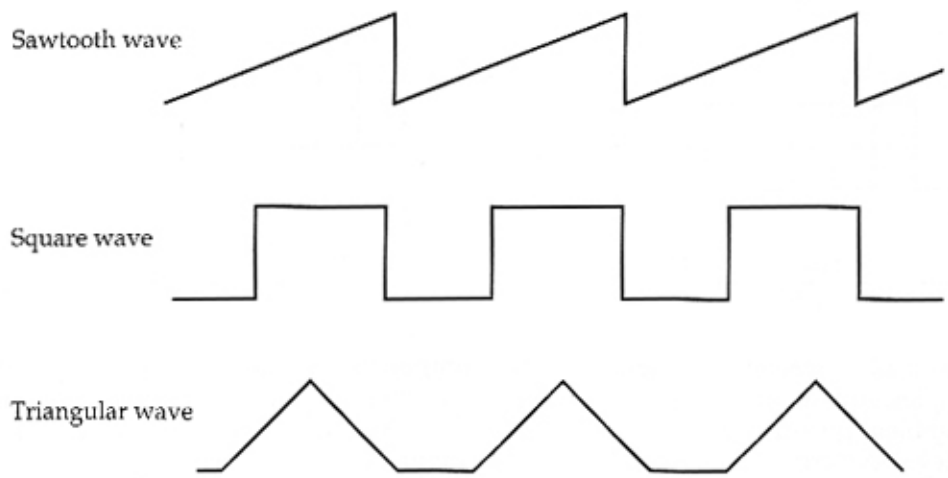
Frequency modulation (FM)



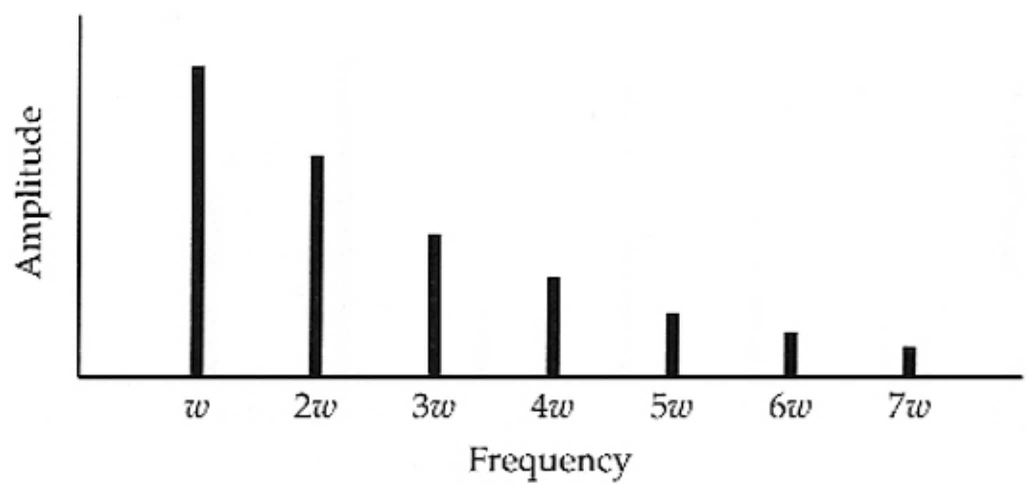
Modulation determines the number of side bands



Periodic nonsinusoidal signals

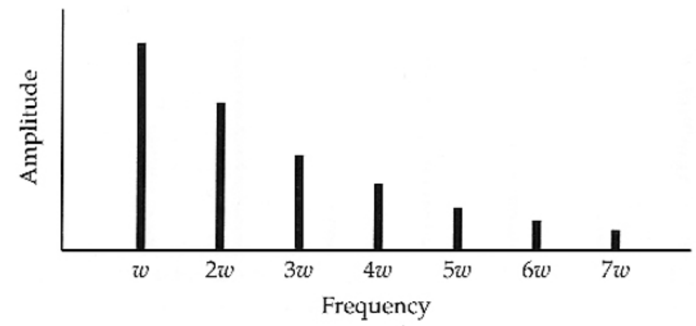


Harmonic series



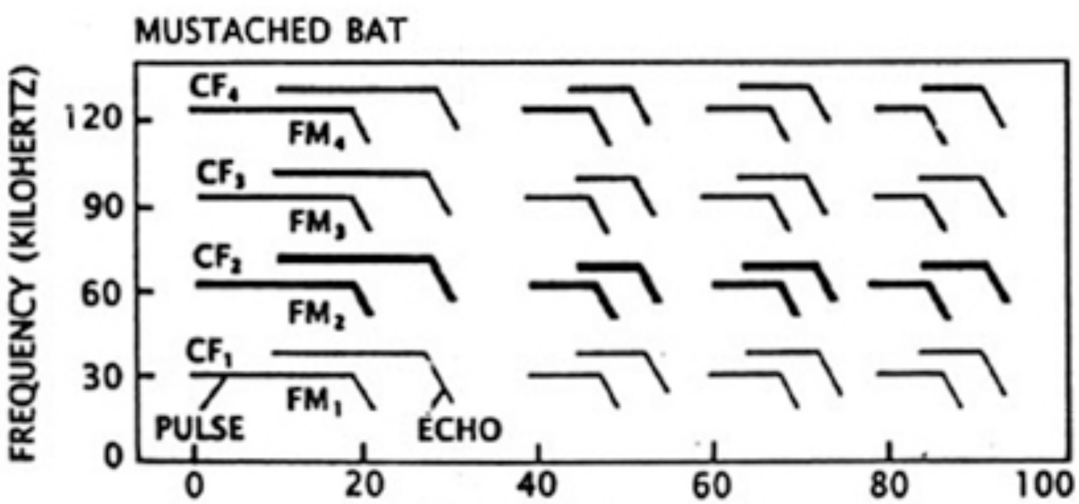
Harmonic series

- Harmonic frequencies are integer multiples of the fundamental frequency, i.e. w, 2w, 3w, 4w …

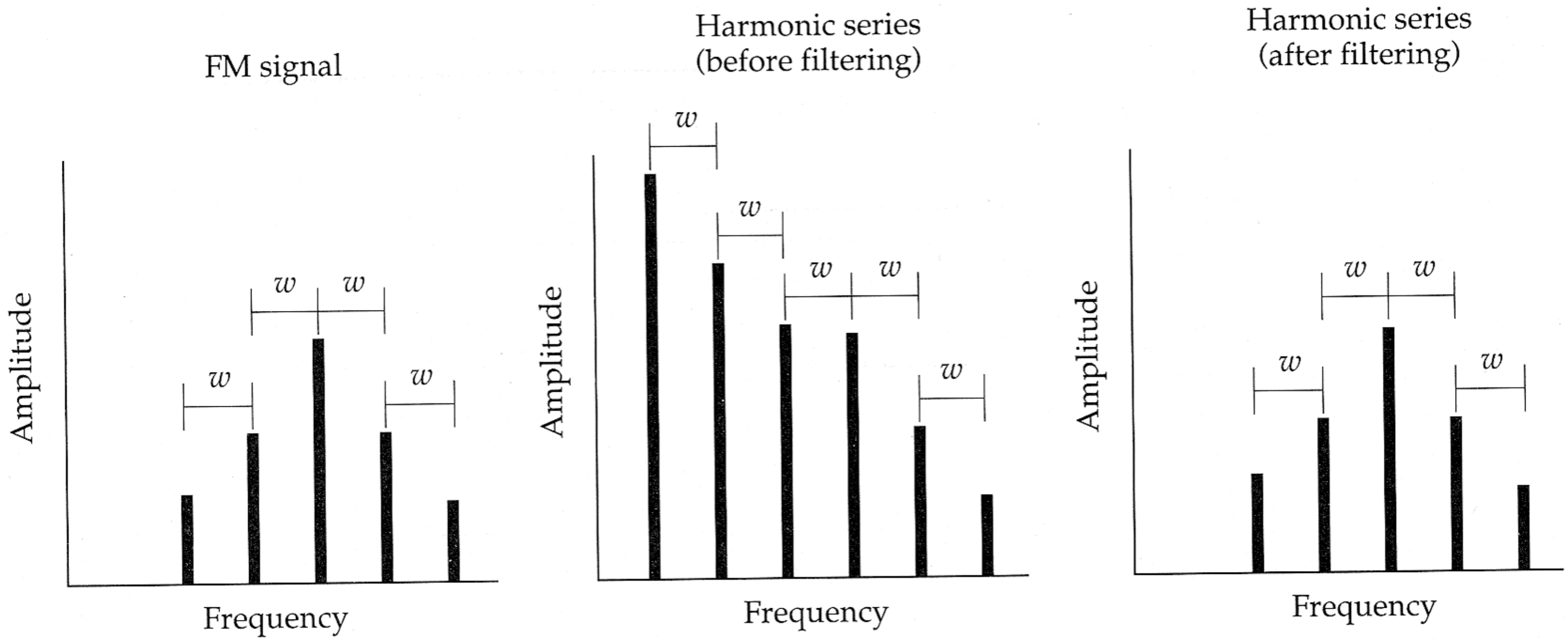


- Dirichlet's rule states that the energy in higher harmonics falls off exponentially with the frequency of the harmonic
- Note, however, that some animals alter the amplitude of harmonics by selective filtering during sound production

Harmonic filtering example



Harmonic Filtering



Frequency modulated sounds and periodic nonsinusoidal sounds can have identical spectra, but will differ in phase

Compound signals

- Nonsinusoidal modulation of a sine wave
- Sinusoidal modulation of a nonsinusoidal carrier wave
- Nonsinusoidal modulation of a nonsinusoidal carrier wave

Pulsed sine wave (frog or insect)

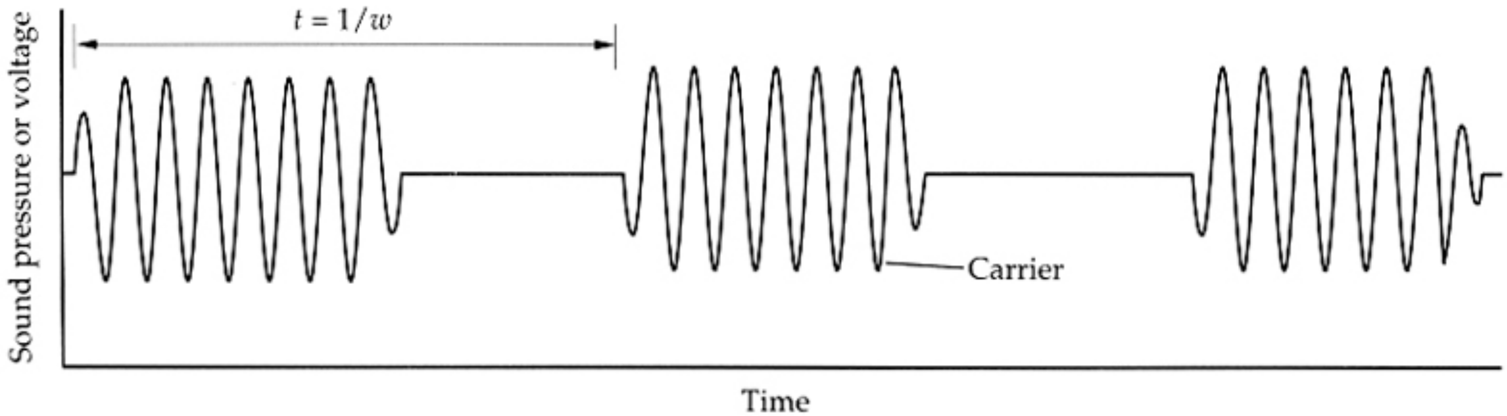
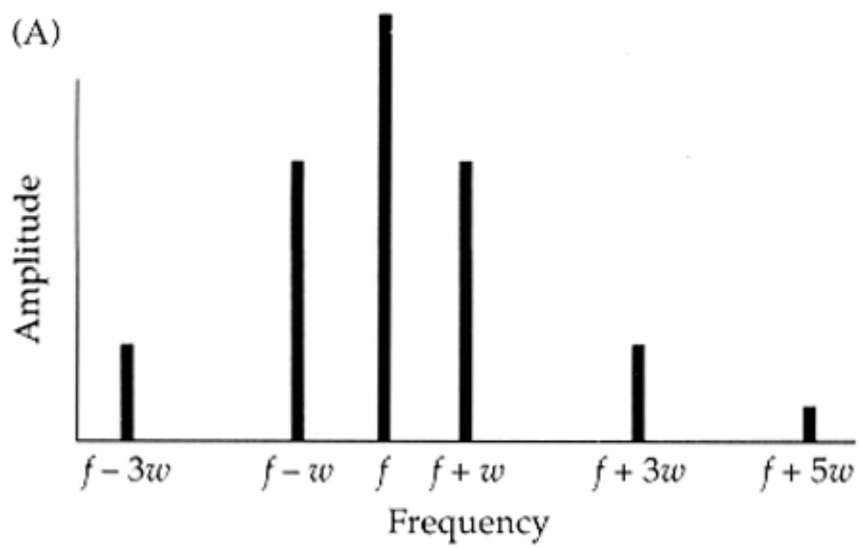


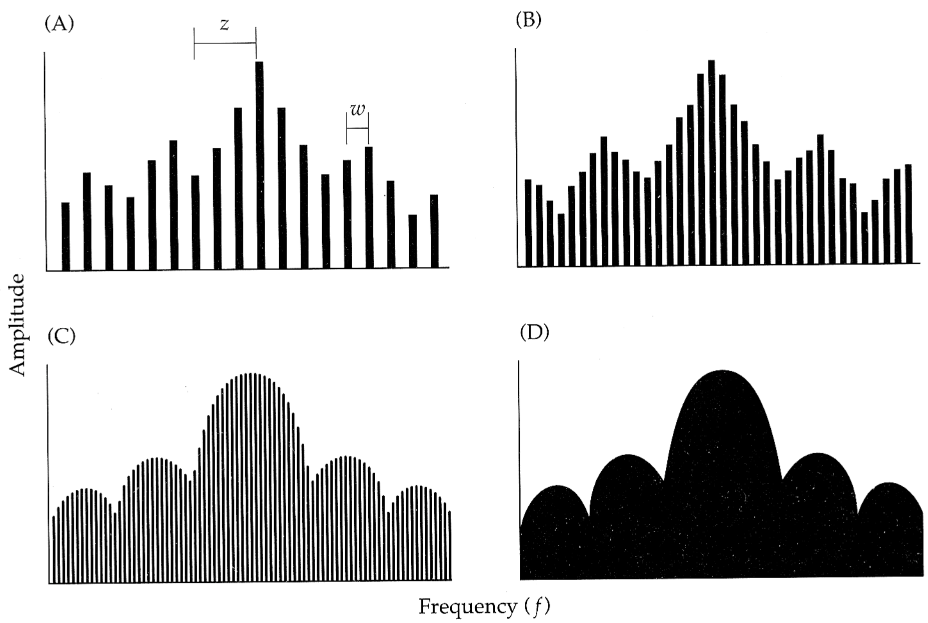
Figure 3.15 Time domain image of a pulsed sine wave. The carrier here is a pure sine wave of frequency f. The modulating waveform is a square wave with frequency w.



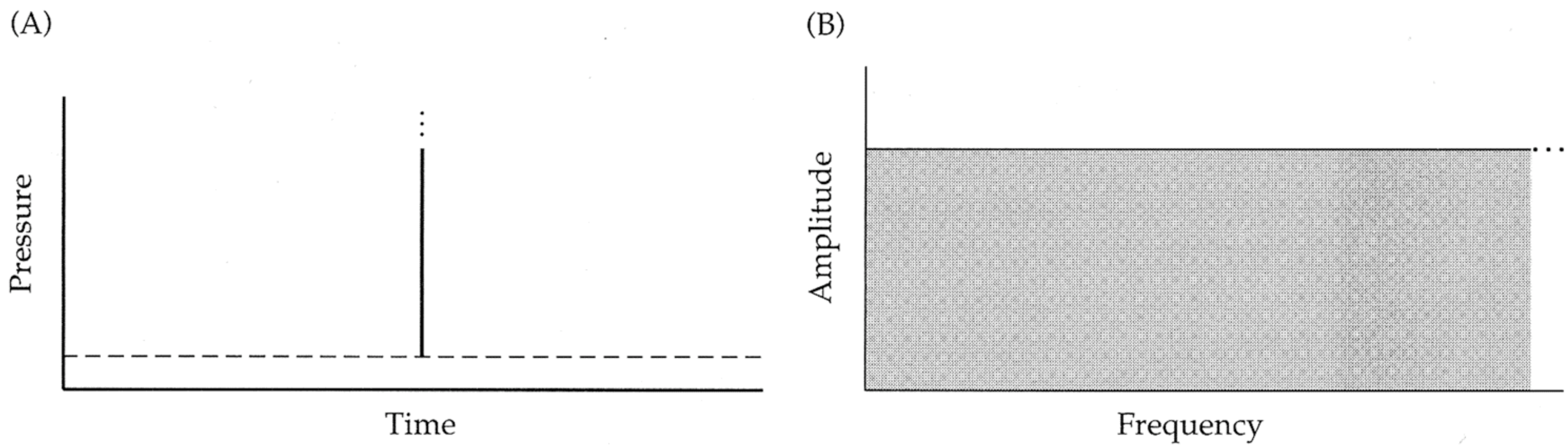
Fourier analysis of aperiodic signals

Most natural signals have a short duration, not infinite

- The more aperiodic a signal is, the more frequency components are needed to describe the signal with a Fourier series
- A - D have increasing intervals, between pulses, each of which has carrier frequency f. D is a single pulse

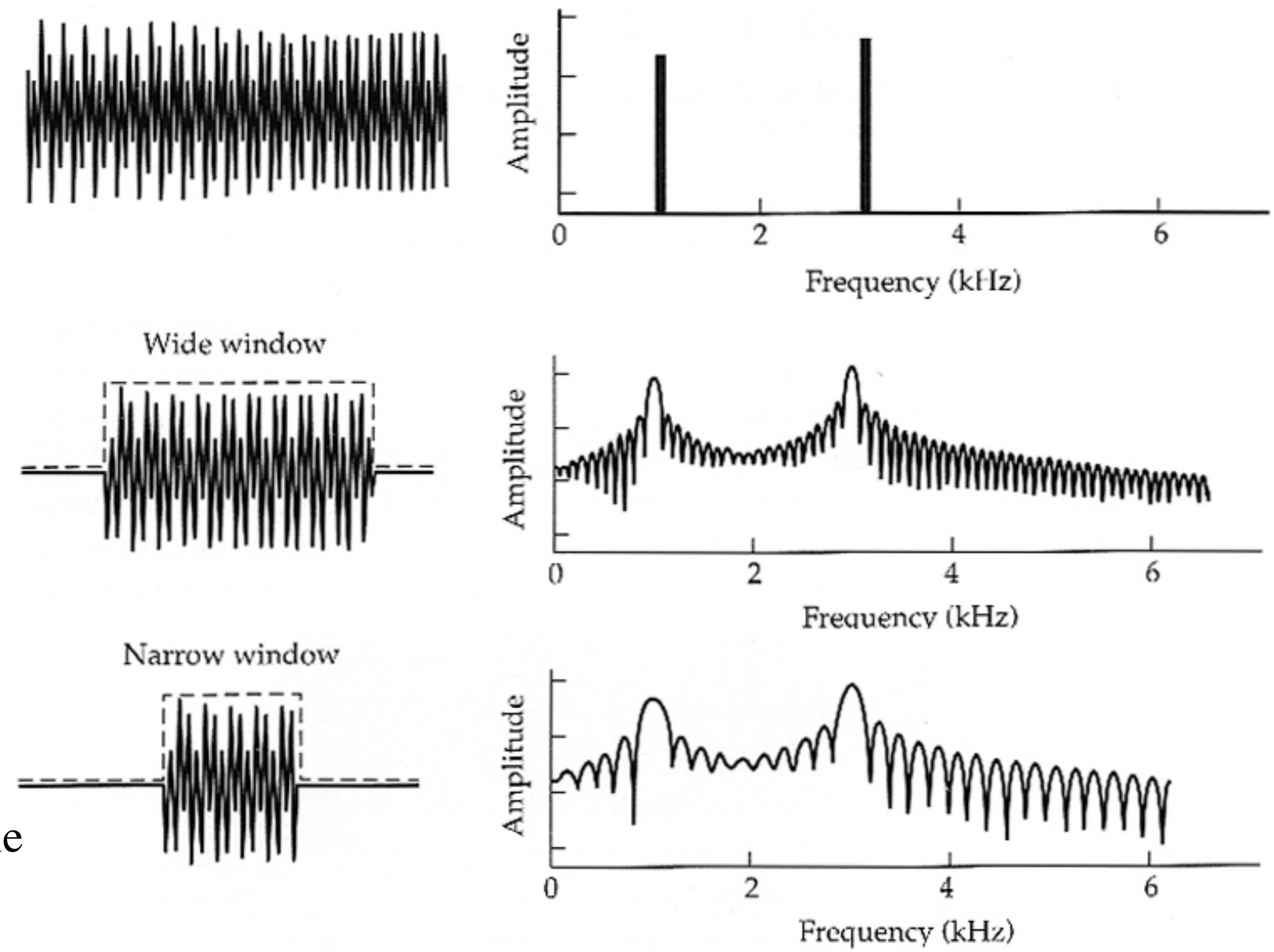


Delta signals



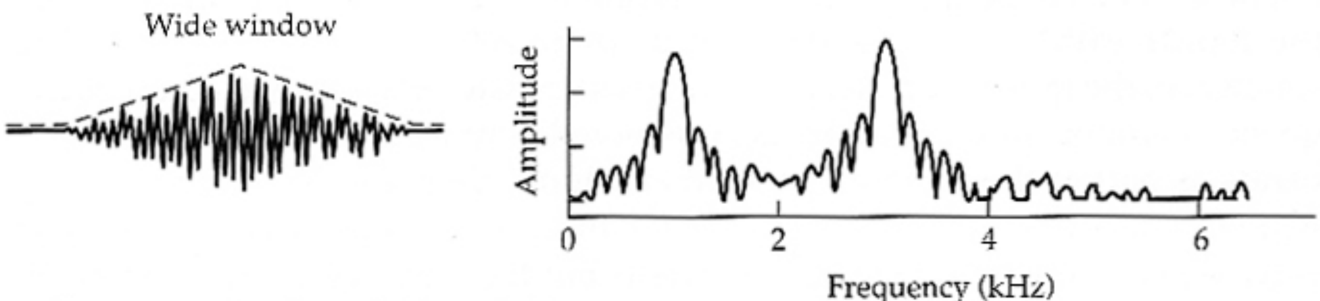
- In the limit, an infinitely short signal has constant amplitude at all frequencies, a delta pulse
- White noise also has equal amplitude at all frequencies, but this results from random waveform

Finite sounds and Fourier 'lobes'

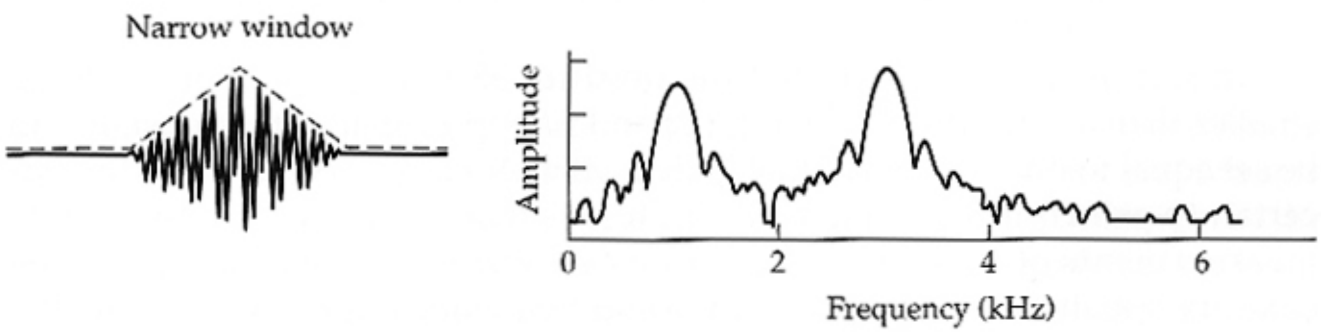


Lobes are artifacts of short sample

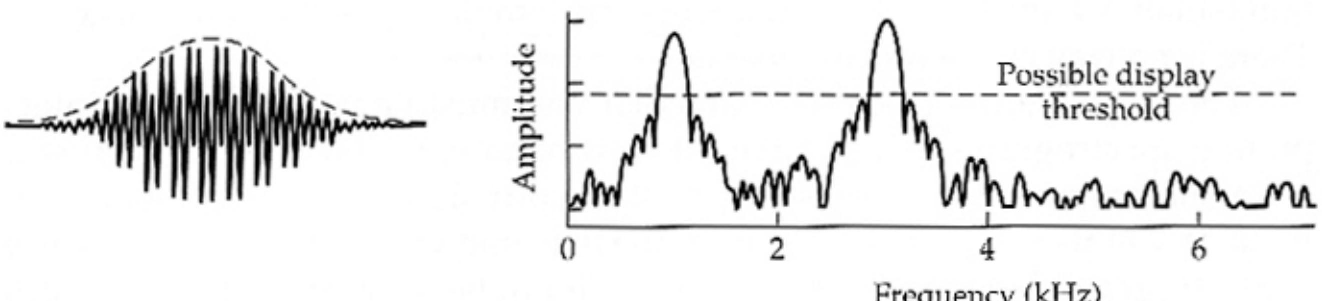
Fourier “windows”



Bartlett and a narrow Bartlett window yields:



Hamming



Digitizing sound

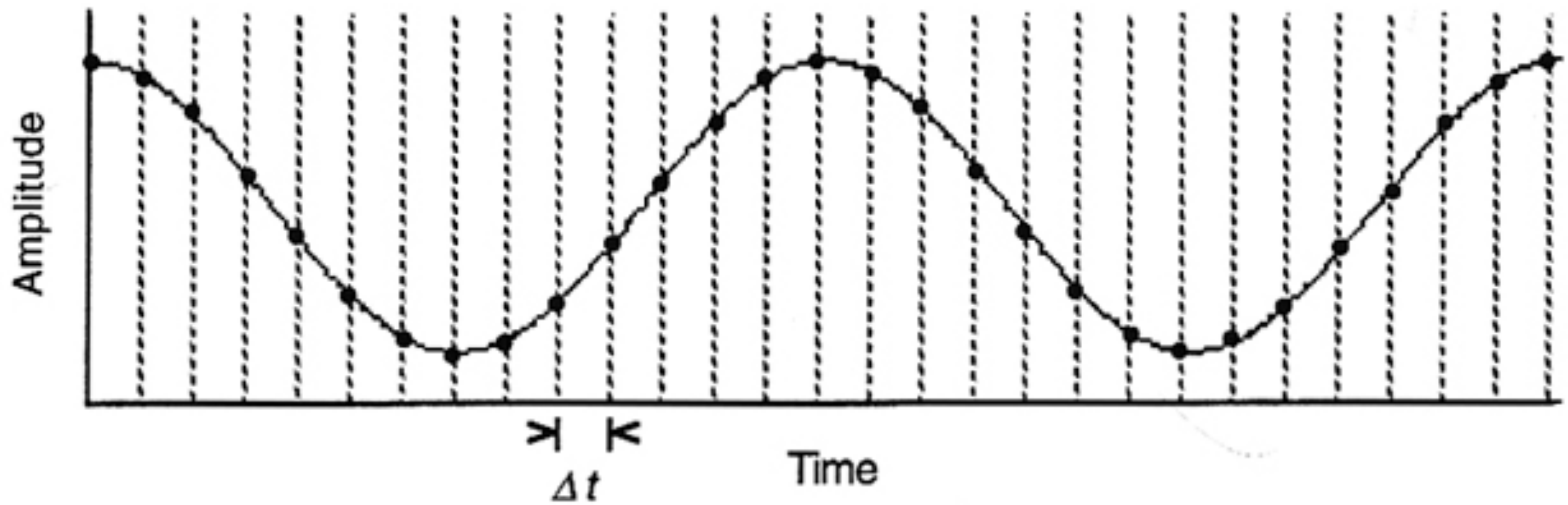


Figure A.1. Sampling to create digital representation of a pure tone signal. Measurements of the instantaneous amplitude of the signal are taken at a sampling rate of 1/Δt. The resulting sequence of amplitude values is the digitized signal.

Typical sampling rates are 22 kHz and 44 kHz

Aliasing

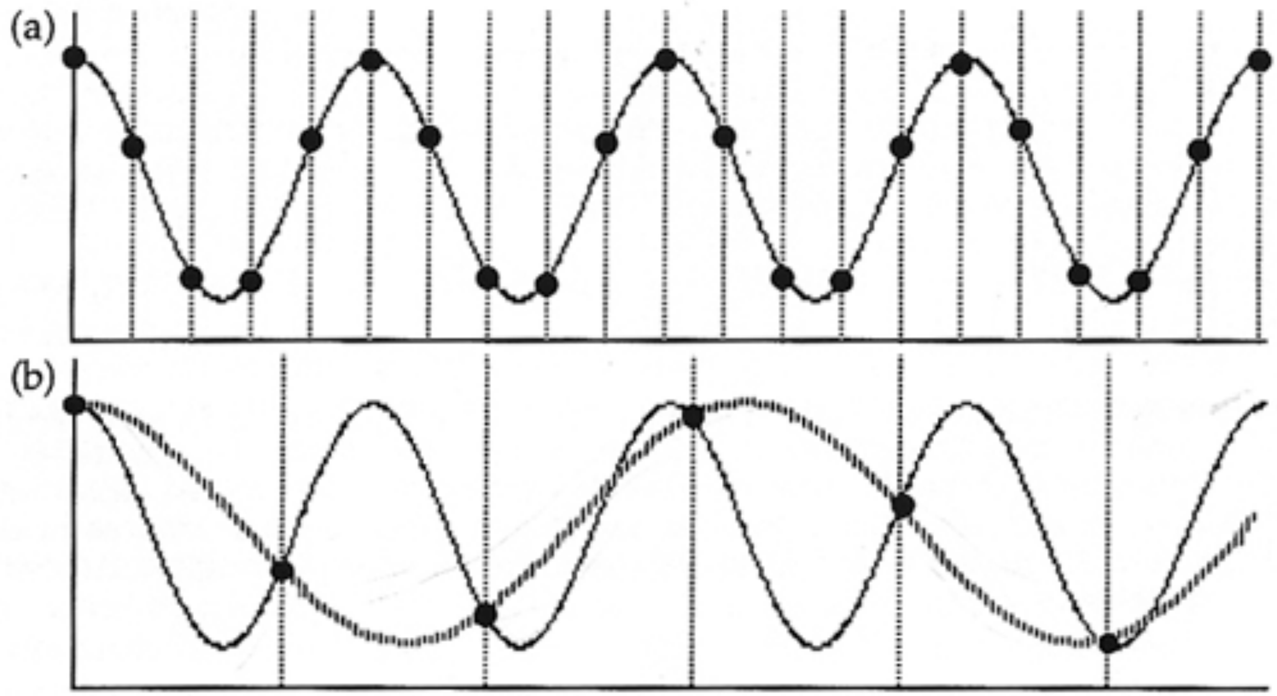
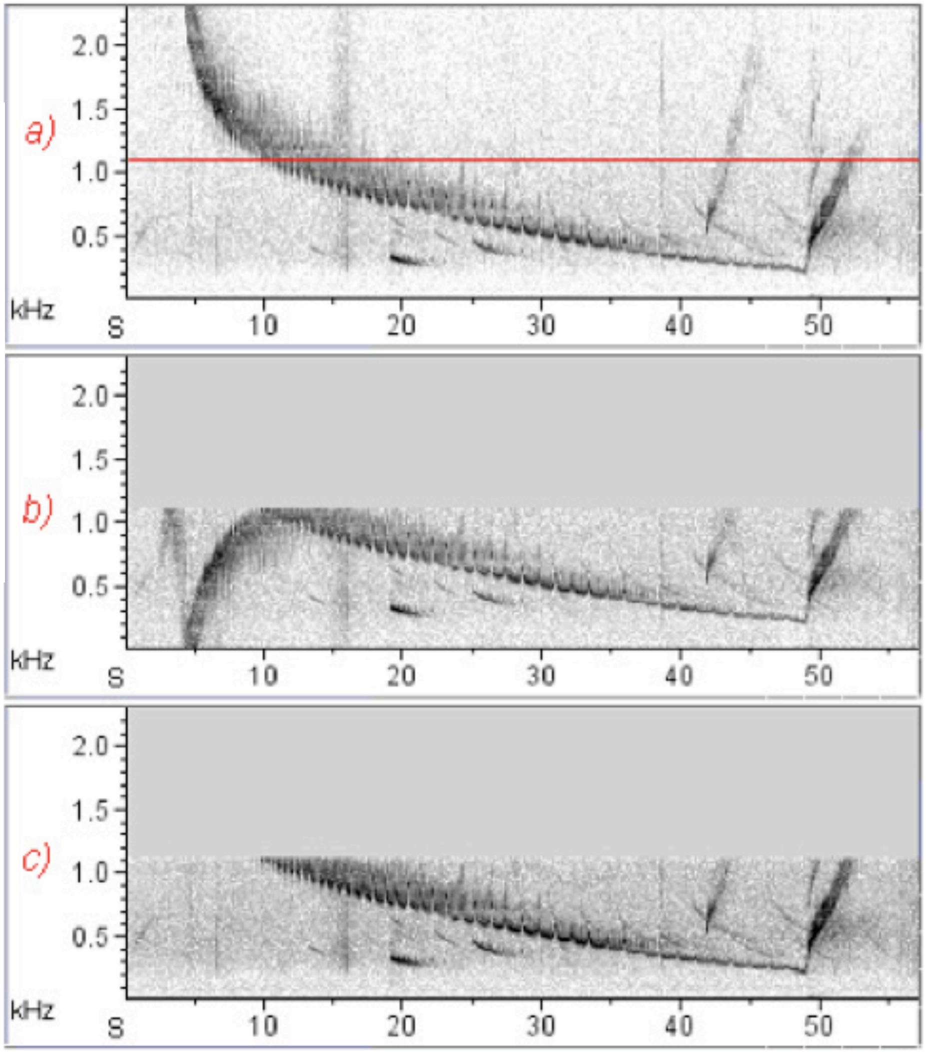


Figure A.2. Aliasing as a result of inadequate sample rate. The same analog waveform is shown in both figures. Vertical lines indicate times at which samples are taken. **(a)** Sampling frequency approximately five times the signal frequency. **(b)** Sampling frequency approximately 1.5 times the signal frequency. The resulting digitized signal (gray waveform) exhibits aliasing: it portrays a waveform of lower frequency than the original analog signal.

Nyquist frequency = 1/2 the sampling frequency

Aliasing consequences

Figure A.3. Appearance of aliasing in spectrogram views. **(a)** Spectrogram of a bearded seal song signal digitized at 11025 Hz. All of the energy in the signal is below the Nyquist frequency (5512.5 Hz); only the lowest 2300 Hz is shown. The red line is at 1103 Hz, one-fifth of the Nyquist frequency. **(b)** The same signal sampled at 2205 Hz (one-fifth of the original rate; Nyquist frequency, 1102.5 Hz) without an anti-aliasing filter. The frequency downsweep in the first ten seconds of the original signal appears in inverted form in this undersampled signal, due to aliasing. **(c)** The same signal as in (b), but this time passed through a low-pass (anti-aliasing) filter with a cutoff of 1100 Hz before being digitized. The downsweep in the first ten seconds of the original signal, which exceeds the Nyquist frequency, does not appear because it was blocked by the filter.



Fourier window size and bandwidth

Bandwidth = minimum frequency difference that can be discerned

Narrow Bandwidth
(135 Hz, 46 ms)

Wide Bandwidth
(540 Hz, 11.5 ms)

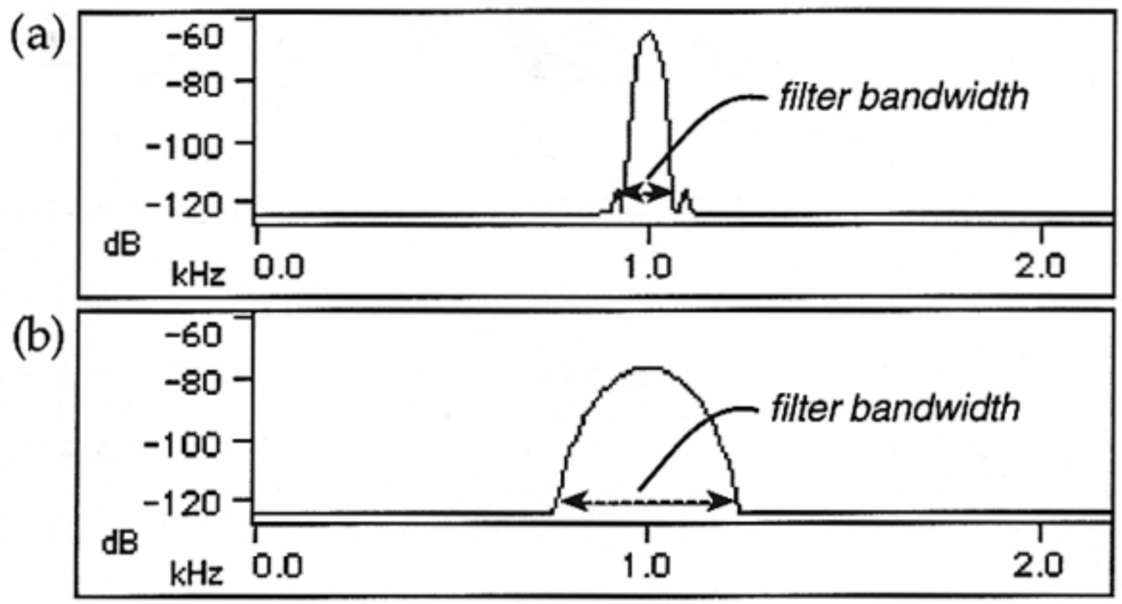
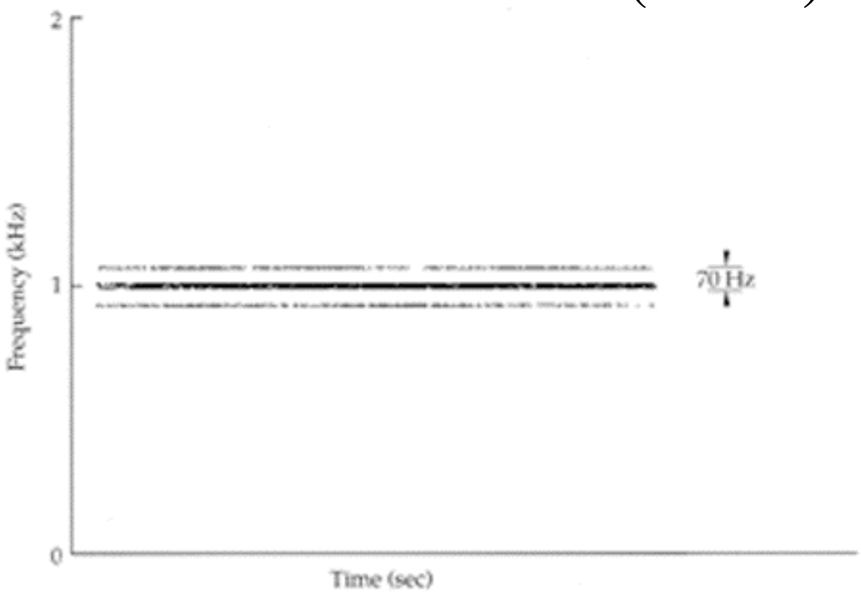


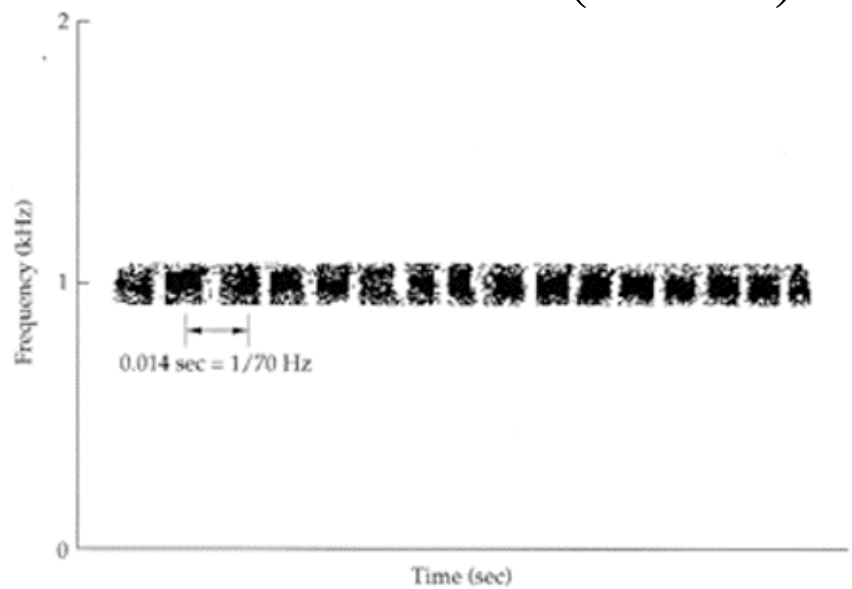
Figure B.5. Relationship between frame length and filter bandwidth.[2] Each spectrum is of a single frame of a 1000 Hz tone, digitized at 22.3 kHz. In both spectra, FFT size = 2048 points, window function = Blackman, clipping level = -130 dB.
(a) Frame length = 1024 points = 46.0 mS; filter bandwidth = 135 Hz.
(b) Frame length = 256 points = 11.5 mS; filter bandwidth = 540 Hz.

Narrow band vs wide band spectrogram

Narrow bandwidth (50 Hz)



Wide bandwidth (300 Hz)



1 kHz signal sinusoidally amplitude modulated at 70 Hz

- Narrow bandwidth analysis is good for frequency
- Wide bandwidth is good for time

Time-frequency uncertainty

Wide Bandwidth
(1412 Hz, 2.9 ms)

Narrow Bandwidth
(176 Hz, 23 ms)

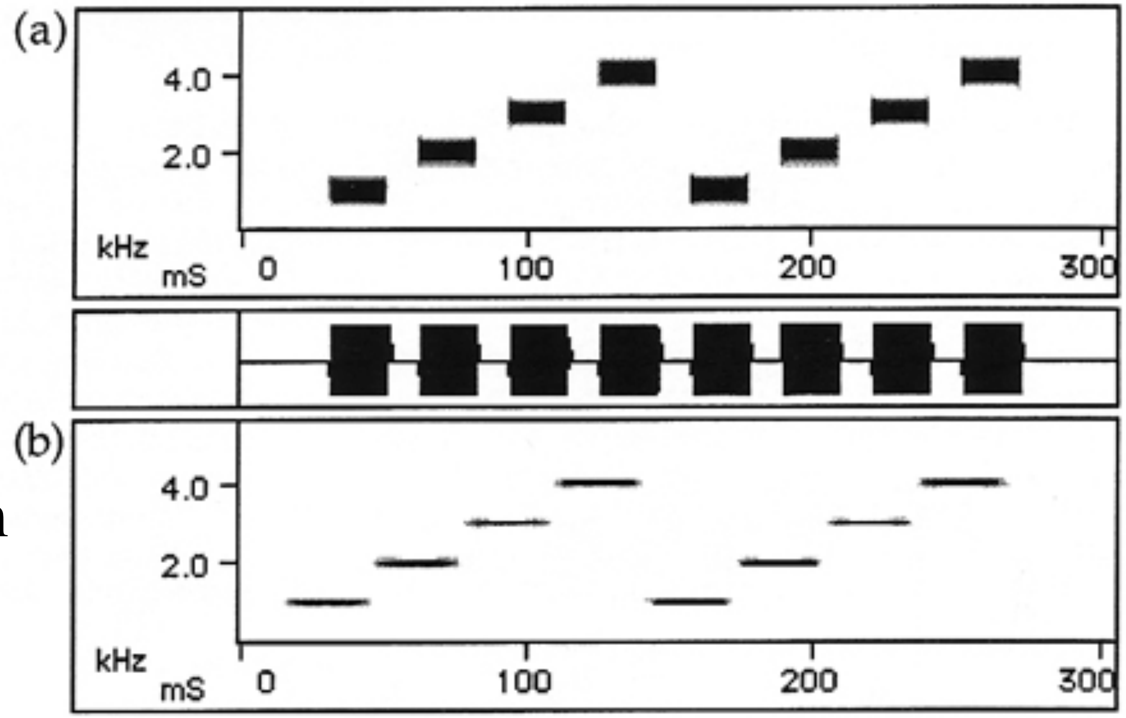


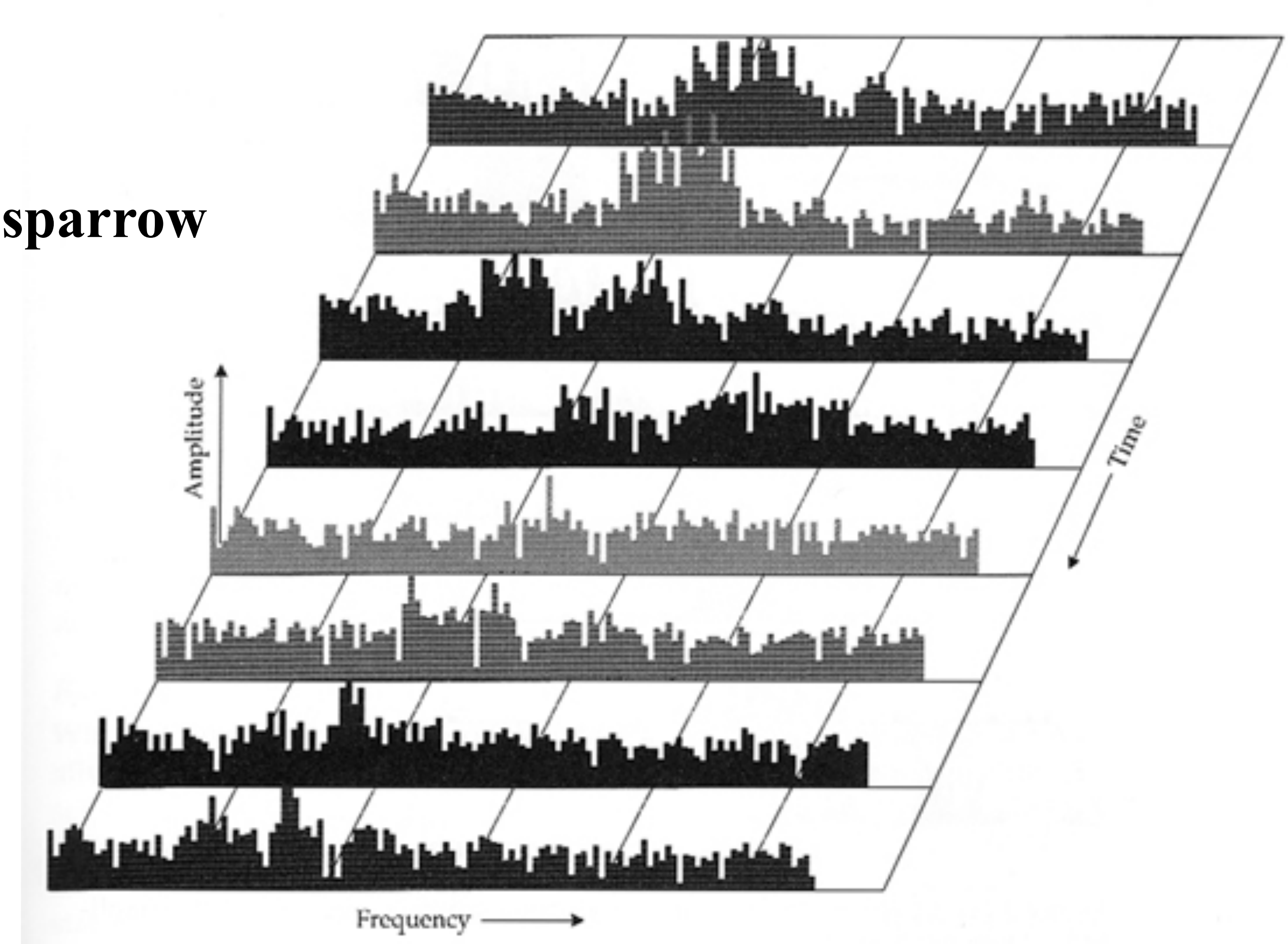
Figure B.8. Effect of frame length and filter bandwidth on time and frequency resolution. The signal consists of two repetitions of a sequence of four tones with frequencies of 1, 2, 3, and 4 kHz. Each tone is 20 mS in duration. The interval between tones is 10 mS. For both spectrograms, time grid resolution = 1.4 mS, frequency grid resolution = 43.5 Hz (FFT size = 512 points), window function = Hamming, clipping level = -100 dB.
(a) Frame length = 64 points (= 2.9 mS), filter bandwidth = 1412 Hz
(b) Frame length = 512 points (= 23.0 mS), filter bandwidth = 176 Hz.
The waveform between the spectrograms shows the timing of the pulses.

Problems

- If you want to measure the time delay between an echolocation pulse and the returning echo, should you use a 64 or 1024 point window?
- If you want to measure the frequency of a constant frequency echolocation pulse accurately, should you use a 64 or 1024 point window?

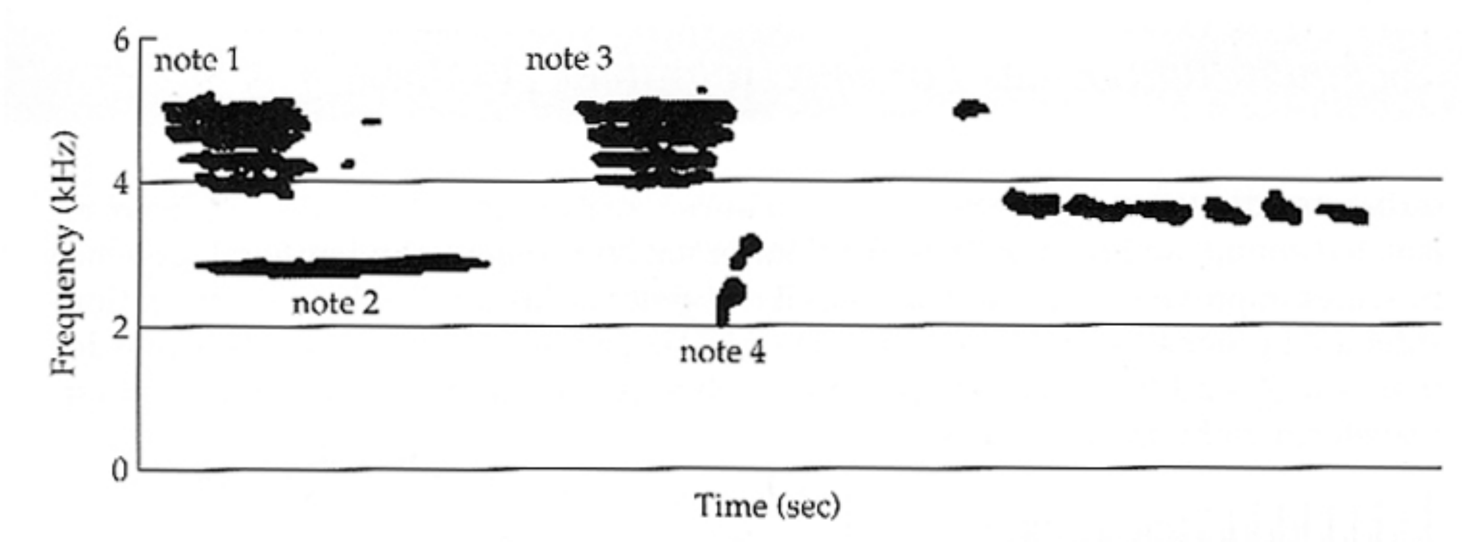
Sound spectrum “waterfall”

Song sparrow



Sonagram = sound spectrogram

Song sparrow



Narrow bandwidth analysis is good for frequency measurements, but not accurate for time measurements